

# Thermal performance of nanofluid flow in microchannels

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## Abstract

Two effective thermal conductivity models for nanofluids were compared in detail, where the new KKL (Koo–Kleinstreuer–Li) model, based on Brownian motion induced micro-mixing, achieved good agreements with the currently available experimental data sets. Employing the commercial Navier–Stokes solver CFX-10 (Ansys Inc., Canonsburg, PA) and user-supplied pre- and post-processing software, the thermal performance of nanofluid flow in a trapezoidal microchannel was analyzed using pure water as well as a nanofluid, i.e., CuO–water, with volume fractions of 1% and 4% CuO-particles with  $d_p = 28.6$  nm. The results show that nanofluids do measurably enhance the thermal performance of microchannel mixture flow with a small increase in pumping power. Specifically, the thermal performance increases with volume fraction; but, the extra pressure drop, or pumping power, will somewhat decrease the beneficial effects. Microchannel heat sinks with nanofluids are expected to be good candidates for the next generation of cooling devices.

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**Keywords:** Brownian motion; Micro-mixing; Effective thermal conductivity; Nanofluid flow; Microchannel heat sinks

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## 1. Introduction

Microscale cooling devices, such as microchannel heat sinks, are increasingly important in current and future heat removal applications. Specifically, a coolant flowing through a large number of parallel, micromachined or etched conduits with the purpose to remove heat from and generate uniform temperature distributions in micro-electro-mechanical systems, integrated circuit boards, laser-diode arrays, high-energy mirrors and other compact products with high transient thermal loads. Key is the very large heat transfer surface-to-volume ratio of the devices, leading to high compactness and effectiveness of heat removal. Complementary to that is the use of high thermal performance coolants. Most exciting are new coolants consisting of a combination of a low-volume fraction of nanoparticles (e.g., metals, metal-oxides, or carbon-based material) with a suitable carrier fluid, such as distilled water, engine oil, eth-

ylene glycol. As first demonstrated in 1995 at Argonne National Laboratory, those dilute liquid-particle mixtures, called nanofluids, have exhibited thermal conductivity values 20–150% higher than the ones of the base fluids (Choi, 1995; Chopkar et al., 2007). Over the last seven years numerous experimental and a few theoretical papers appeared, providing nanofluid property measurements and models describing the underlying physics of enhanced thermal conductivities for different nanoparticle-and-liquid pairings.

The effective thermal conductivity ( $k_{\text{eff}}$ ) of any nanofluid depends mainly on the nanoparticle volume fraction, conductivity and diameter, as well as the carrier-fluid temperature and conductivity. Fig. 1 provides a glimpse of the most recent measurements for  $k_{\text{eff}}$  vs.  $\varphi$  (the volume fraction), considering different temperatures, types of mixtures, and nanoparticle diameters. Metal-oxides (e.g.,  $\text{Al}_2\text{O}_3$ ,  $\text{CuO}$ ,  $\text{TiO}_2$ , etc.) were first used in static nanofluid studies (see Masuda et al., 1993; Lee et al., 1999; Zhou and Wang, 2002; Xie et al., 2002; and Chang et al., 2005; among others). However, nanofluids with metals (e.g., Cu, Fe, Au, etc.) or carbon-based materials generated higher  $k_{\text{eff}}$  values.

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## Nomenclature

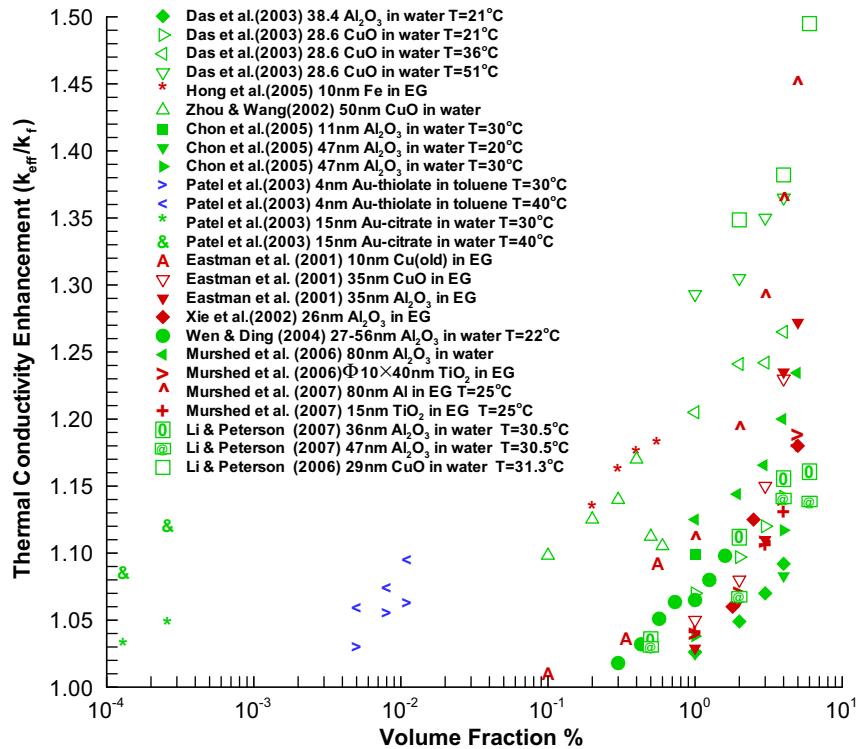
$A_{\text{af}}$	aspect factor ( $=H/W_b$ )	$V_N$	root-mean-square velocity
$A_{\text{ar}}$	aspect ratio ( $=W_b/W_t$ )	$W_b$	channel bottom width
$A_c$	area of the channel cross section	$W_t$	channel top width
$A_{\text{bottom}}$	area of the heated substrate wall		
$C$	constant		
$C_f$	friction coefficient		
$c_p$	specific heat capacity		
$d$	diameter		
$D_h$	hydraulic diameter		
$H$	channel height		
$k$	thermal conductivity		
$L$	channel length		
$m$	constant		
$Nu$	Nusselt number		
$p$	pressure		
$P$	pumping power		
$Pr$	Prandtl number		
$q''$	uniform heat flux		
$Q$	volumetric flow rate		
$R_b$	interfacial thermal resistance		
$Re$	Reynolds number		
$T$	temperature		
$u, v$	velocity		
			<i>Greek Letters</i>
		$\alpha$	nanoparticle Biot number
		$\varphi$	volume fraction
		$\kappa_b$	Boltzmann constant
		$\mu$	dynamic viscosity
		$\nu$	kinematic viscosity
		$\theta$	thermal resistance
		$\rho$	density
			<i>Subscripts</i>
		ave	average
		eff	effective
		f	base fluid
		in	inlet
		int	interface
		m	mean
		out	outlet
		p	particle
		w	wall

For example, [Eastman et al. \(2001\)](#) observed that a nanofluid consisting of 10 nm copper nanoparticles dispersed in ethylene glycol (EG) had a much higher effective thermal conductivity than either pure EG or EG containing the same volume fraction of dispersed metal-oxide nanoparticles. [Choi et al. \(2003\)](#) measured a 300% enhancement of thermal conductivity with 3 wt% loading of single-wall carbon nanotubes. The temperature dependence of  $k_{\text{eff}}$  for the polymer composites was also demonstrated in their experiments. In order to avoid aggregates, these carbon nanotubes have to be surface-treated, or, say, nitric acid has to be added to the mixture. In general, nanofluids overcome the drawbacks of liquids with large particles, i.e., rapid particle sedimentation, clogged flow channels, eroded conduits, and elevated pressure drops.

Although the experimental results have demonstrated a significant potential for thermal conductivity enhancement, the determination of the underlying physics is still in a primary stage, complicated by the fact that the available experimental data from different research group may vary significantly (see [Koo and Kleinstreuer, 2003](#); among others). Clearly, nanoparticle volume fraction, physical characteristics and fluid type are important factors, but insufficient to explain the anomalous increase in thermal performance of nanofluids. Other reasons proposed include a very high particle-liquid interface conductivity due to molecular-size layering of the carrier fluid around the particles as well as Brownian motion of the nanoparticles causing micro-mixing. Additional mechanisms include the effect

of nanoparticle clustering, the nature of heat transfer inside the particles, the interaction and collision among particles, thermal waves via hyperbolic heat conduction, and nanoparticle dispersion. Other nanoparticle-motion mechanisms, such as thermophoresis and osmophoresis, are negligible (Koo and Kleinstreuer, 2005a,b). Based on a molecular dynamics simulation, Sarkar and Selvam (2007) concluded that the thermal transport enhancement of nanofluids was mostly due to the increased movement of liquid atoms in the presence of nanoparticles. However, some researchers questioned the role of Brownian motion hydrodynamics for the unusual thermal effect of nanofluid (see Evans et al., 2006; Vladkov and Barrat, 2006; among others) as well as the actual  $k_{\text{eff}}$ -increase as reported in experimental papers (see Venerus et al., 2006; Putnam et al., 2006; Beck et al., 2007; among others).

Nevertheless, the hypothesis that Brownian motion of the nanoparticles causes micro-mixing has recently become more and more popular (see Koo and Kleinstreuer, 2004, 2005a,b; Jang and Choi, 2004, 2006, 2007; Prasher et al., 2006; among others). In a comparison study where Kleinstreuer and Li (in press) discussed the effective thermal conductivity theory of Jang and Choi (2004, 2006, 2007), it was found that their model cannot consistently match measured thermal performances of nanofluids, especially when the fluid temperature changes. Thus, to predict the thermal performance of nanofluids in microscale cooling devices and other thermal micro-systems, the influence of temperature on the effective thermal conductivity has to be considered.

Fig. 1. Sample of experimental evidence for elevated  $k_{\text{eff}}/k_f$  of nanofluids.

## 2. Theory and analysis

Of interest is the use of a most suitable  $k_{\text{eff}}$  model for the computational analysis of nanofluid flow in a representative microchannel of a typical cooling device. Hence, first two competitive  $k_{\text{eff}}$  models are compared in light of benchmark experimental data sets. Then, pure water and nanofluid flow with heat transfer is investigated.

### 2.1. Temperature-dependent $k_{\text{eff}}$ models for nanofluids

One possible  $k_{\text{eff}}$  correlation proposed by Prasher et al. (2006) is the multi-sphere Brownian model (MSBM) which can be described as follows. After having analyzed several possible thermal conduction mechanisms and associated models, Prasher et al. (2006) confirmed that the localized convection in the liquid due to Brownian movement of the particles is primarily responsible for the observed enhancement of the effective thermal conductivity of nanofluids. They captured random dispersions with a “Brownian-motion Reynolds number” based on the root-mean-square velocity of a Brownian particle as part of a modified Maxwell-Garnett thermal conductivity model (Nan et al., 1997), which also considers the influence of interfacial thermal resistance,  $R_b$ , between nanoparticles and different fluids. The description of multi-sphere convective interaction was borrowed from fluidized bed heat transfer (Brodkey et al., 1991). Combining these phenomena, the MSBM was proposed in the form:

$$\frac{k_{\text{eff}}}{k_f} = (1 + C Re^m Pr^{0.333} \varphi) \times \left( \frac{[k_p(1 + 2\alpha) + 2k_m] + 2\varphi[k_p(1 - 2\alpha) - k_m]}{[k_p(1 + 2\alpha) + 2k_m] - \varphi[k_p(1 - \alpha) - k_m]} \right) \quad (1)$$

where  $Re = V_N d_p / v$ ,  $V_N$  is the root-mean-square velocity,  $v$  is the kinematic viscosity of the liquid,  $\alpha = 2R_b k_m / d_p$  is the nanoparticle Biot number,  $R_b$  is the selective interfacial thermal resistance,  $k_m$  is the medium thermal conductivity, and  $C$  and  $m$  are adjustable constants; while  $C$  is independent of the fluid type,  $m$  depends on the fluid. For example, comparing the MSBM with data from water-based nanofluids and assuming  $R_b$  to be  $0.77 \times 10^{-8} \text{ km}^2 \text{ W}^{-1}$ , they calculated  $C = 40,000$  and  $m = 2.5 \pm 0.15$ . Ethylene glycol (EG) and engine oil were also matched well when employing appropriate  $R_b$  and  $m$  values. As shown in Fig. 2, the MSBM model generates a good agreement with some of the recent  $\text{Al}_2\text{O}_3$ -water experimental data sets when employing a decent  $m$  value. However, the MSBM model can not predict the thermal conductivity enhancement trend for the experimental results of Li and Peterson (2007). Interestingly enough, the experimental data of Chon et al. (2005) also indicated a different trend while employing the same nanofluids, i.e., the same particle diameter and volume fraction. The MSBM comparisons were also not convincing when the particle was too small ( $\leq 11 \text{ nm}$ ) or too large ( $\geq 125 \text{ nm}$ ).

According to the postulate that submicron particle Brownian motion has explicitly a significant impact on the effective thermal conductivity, Koo and Kleinstreuer

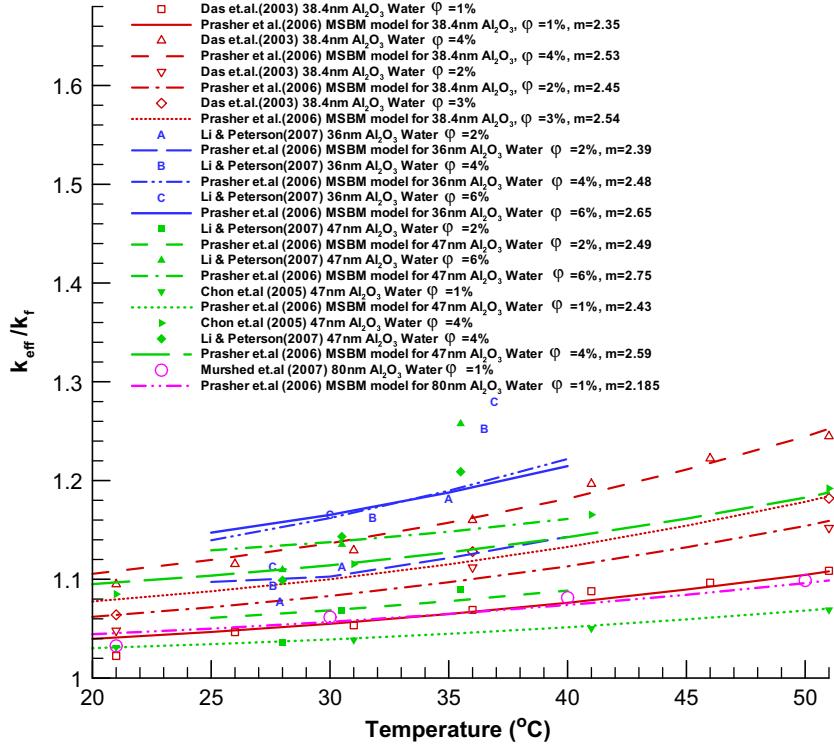


Fig. 2. Comparison of Prasher et al. (2006) thermal conductivity enhancement model (MSBM) with experimental data for  $\text{Al}_2\text{O}_3$ –water nanofluids at different temperatures.

(2004) proposed that  $k_{\text{eff}}$  is composed of the nanoparticle's conventional static part and a Brownian motion part. Their thermal conductivity model takes into account the effects of particle size, particle volume fraction and temperature dependence as well as type of nanoparticle and base fluid combinations. Specially,

$$k_{\text{eff}} = k_{\text{static}} + k_{\text{Brownian}} \quad (2)$$

where the static part is Maxwell's model and the dynamic part was developed based on kinetic theory together with Stokes' flow of micro-scale convective heat transfer, i.e., *micro-mixing*. Hence,

$$\frac{k_{\text{static}}}{k_f} = 1 + \frac{3 \left( \frac{k_p}{k_f} - 1 \right) \varphi}{\left( \frac{k_p}{k_f} + 2 \right) - \left( \frac{k_p}{k_f} - 1 \right) \varphi} \quad (3)$$

and

$$k_{\text{Brownian}} = 5 \times 10^4 \beta \varphi \rho_f c_{p,f} \sqrt{\frac{k_b T}{\rho_p d_p}} f(T, \varphi) \quad (4)$$

The functions  $\beta$  and  $f$ , to be determined semi-empirically, were introduced to encapsulate the thermo-hydrodynamic interactions among micro-scale fluid parcels and the nanoparticle interactions capturing any temperature dependence (Koo and Kleinstreuer, 2005a,b). What is missing in Eq. (4) is the interfacial thermal resistance  $R_b$  between nanoparticles and base fluids (see Prasher et al., 2006; Jang and Choi, 2007; Xuan et al., 2006; Xue, 2006). The thermal interfacial resistance (also called Kap-

itz resistance) is believed to exist in the adjacent layers of the two different materials, i.e., the thin barrier layer plays a key role in weakening the effective thermal conductivity of the nanoparticles. For example, Wilson et al. (2002) reported that the magnitude of  $R_b$  between different nanoparticles and base fluids ranges from about  $0.77 \times 10^{-8} \text{ km}^2 \text{ W}^{-1}$  to approximately  $20 \times 10^{-8} \text{ km}^2 \text{ W}^{-1}$ . More recently, Huxtable et al. (2003) showed that the interface thermal resistance across a carbon nanotube and base fluid is  $8.33 \times 10^{-8} \text{ km}^2 \text{ W}^{-1}$ . Choosing an average value of  $R_b = 4 \times 10^{-8} \text{ km}^2 \text{ W}^{-1}$ , the original K & K model was enhanced. In the static part, an effective nanoparticle thermal conductivity was used to substitute the isolated nanoparticle thermal conductivity, i.e.,

$$R_b + \frac{d_p}{k_p} = \frac{d_p}{k_{p,\text{eff}}} \quad (5)$$

The functions  $\beta$  and  $f$  in Eq. (4) were combined to a new  $g$ -function which considers the influence of multi-particle interaction which depends on particle diameter, temperature and volume fraction. For different base fluids and different nanoparticles, the  $g$ -function should differ; presently, only water-based nanofluids were considered because of the limits in available experimental data sets. For example, for  $\text{Al}_2\text{O}_3$ –water and  $\text{CuO}$ –water nanofluids, the nonlinear  $g$ -function generated  $r^2$  values of 96% and 98%, respectively, using benchmark experimental data sets (Li, 2008)

Fig. 3 depicts a comparison of the new Koo–Kleinstreuer–Li (KKL) model with experimental data for  $\text{Al}_2\text{O}_3$ –water nanofluids at different temperatures. Similar

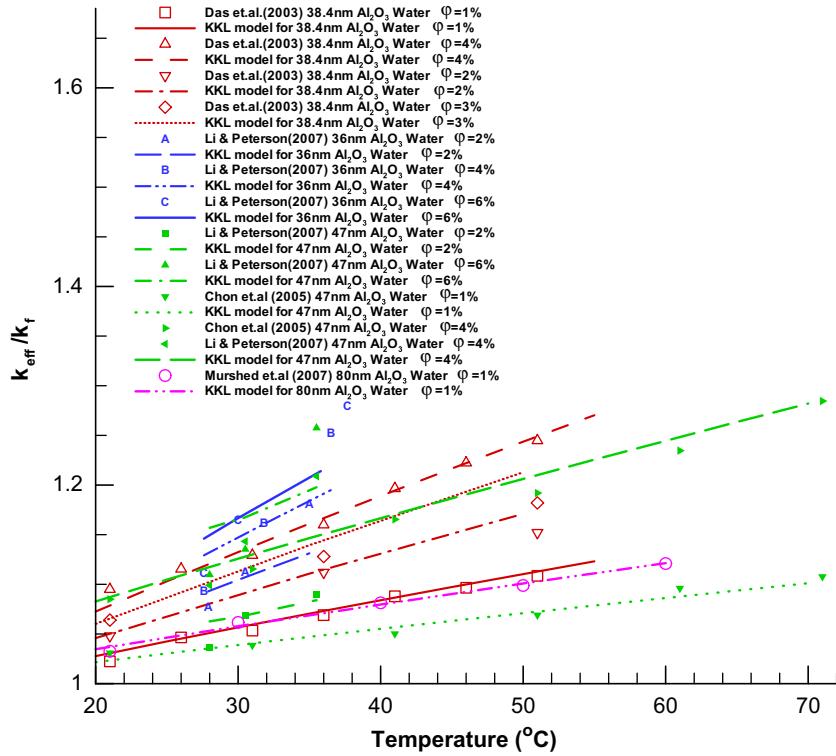


Fig. 3. Comparison of KKL model with experimental data for Al<sub>2</sub>O<sub>3</sub>–water nanofluids at different temperatures.

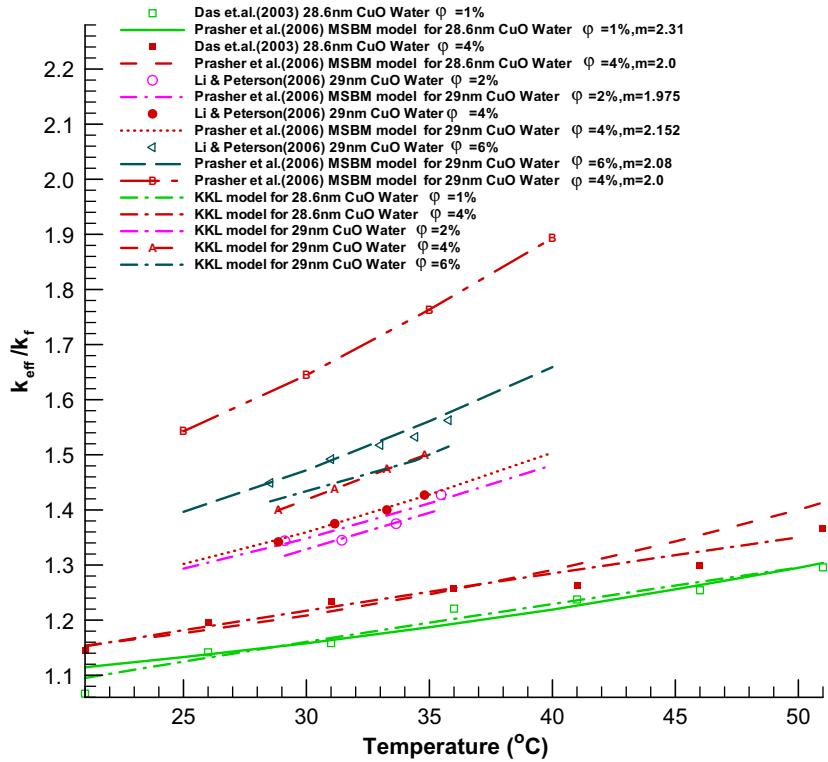


Fig. 4. Comparison of KKL model, MSBM model with experimental data for CuO–water nanofluids at different temperatures.

to the MSBM model, the KKL model shows a very good agreement with most of the recent experimental data sets. In Fig. 4, the KKL and MSBM models are compared with

experimental data for CuO–water nanofluids at different temperatures. For the experimental data of Das et al. (2003), the KKL model provides a better matching than

the MSBM model, while the opposite is the case for the experimental data of Li and Peterson (2006). It must be mentioned that the  $m$ -value has an important influence on the MSBM model. If the value for the Reynolds number exponent is taken to be 2 for the 29 nm 4% CuO–water nanofluids, i.e., the same as for the 28.6 nm 4% CuO–water nanofluids, the predicted  $k_{\text{eff}}$  numbers generated by the MSBM model are far larger than the experiment values.

Both the Koo–Kleinstreuer–Li model and the MSBM model showed good agreements with most experimental results. The KKL model encapsulates micro-mixing due to random nanoparticle motion based on first principles. In contrast, the models by Jang and Choi (2007) and Prasher et al. (2006) contain Brownian motion effects implicitly in terms of “Brownian Reynolds numbers”. Still, in the KKL theory, the complexities caused by multi-sphere interactions and strong temperature changes are lumped into a functional which could be detailed and expanded as more experimental evidence appears. The MSBM model employs adjustable parameters as well as factors  $C$  and  $m$  for data matching.

## 2.2. Pure water flow and nanofluid flow with heat transfer in a trapezoidal microchannel

Considering the strong temperature-dependent characteristics of the nanofluid thermal performance (see Figs. 2–4) and possibly large temperature differences in micro-heat sinks, temperature dependent physical properties of water and the new KKL model for the thermal conductivity of nanofluids were selected. Employing the commercial Navier–Stokes solver CFX-10 (Ansys Inc., Canonsburg, PA) and user-supplied pre- and post-processing software, pure water as well as CuO–water mixtures of different CuO-volume fractions were used as the working fluid.

Specifically, Fig. 5a depicts a representative microchannel as well as the associated computational mesh. The top width  $W_t$ , bottom width  $W_b$  and depth of the channel  $H$  are 500  $\mu\text{m}$ , 358.4  $\mu\text{m}$  and 100  $\mu\text{m}$ , respectively (Chein

and Chuang, 2005). The base angle, which is the angle between the channel side wall and bottom wall, is 54.7°. The length of the channel  $L$  is 27 mm. The heights of the cover and base substrate are both  $H_c = 500 \mu\text{m}$ . A constant heat flux of  $q'' = 431,466 \text{ W/m}^2$  (10 W for the unit element) from below and adiabatic conditions at the other boundaries were assumed (Li et al., 2004). For comparison, first a smooth channel with pure de-ionized water as the working fluid was considered. The hydraulic diameter for the present case is:

$$D_h = \frac{4 \times 0.5 \times (W_t + W_b) \times H}{W_t + W_b + 2 \times H / \sin \theta} = 155.6 \mu\text{m} \quad (6)$$

Other important geometric channel parameters are the aspect ratio as well as the aspect factor, defined respectively as

$$A_{\text{ar}} = W_b / W_t \text{ and } A_{\text{af}} = H / W_b \quad (7a, b)$$

The fluid material properties (density, thermal conductivity, dynamic viscosity, specific thermal capacity at constant pressure) were temperature dependent. For steady-state operations, the continuum mechanics equations are:

$$\nabla \cdot [\rho_f(T) \vec{u}] = 0 \quad (8)$$

$$\rho_f(\vec{u} \cdot \nabla \vec{u}) = -\nabla p + \nabla \cdot (\mu_f \nabla \vec{u}) \quad (9)$$

and

$$\rho_f c_{p,f}(\vec{u} \cdot \nabla T) = k_f \nabla^2 T + \mu_f \Phi \quad (10)$$

where

$$\Phi = \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \frac{\partial u_i}{\partial x_j} \quad (11)$$

Here  $\vec{u}$  is the velocity vector; and  $\Phi$  is the viscous dissipation function. As hydraulic boundary conditions, a uniform velocity is applied at the channel inlet, i.e.,  $u_x = 0$ ,  $u_y = 0$ ,  $u_z = U_{\text{in}}$ ,  $u_x$ ,  $u_y$ , and  $u_z$  are the velocity vector quantities in  $x$ ,  $y$ , and  $z$  direction. The outlet pressure is the static pressure, i.e.,  $p_{\text{out}} = 0$ . The no-slip boundary condition was enforced at all solid walls. The thermal boundary con-

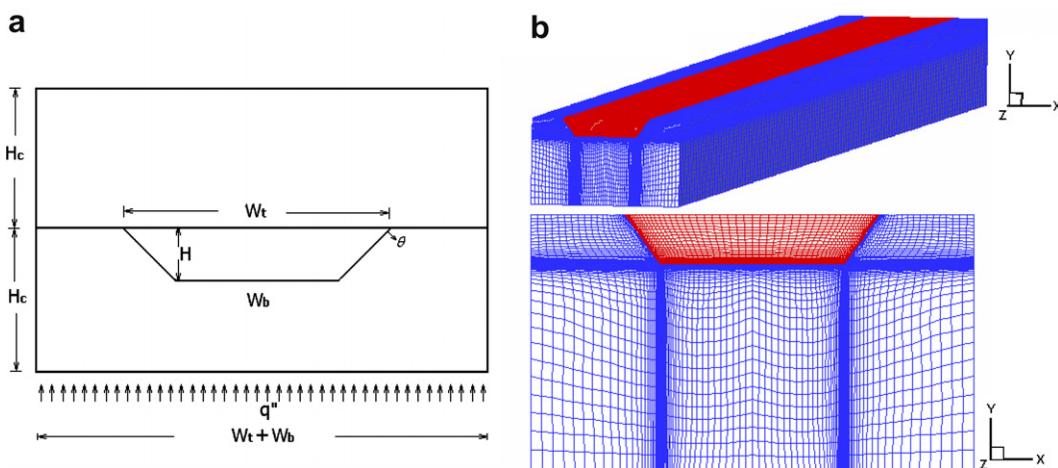


Fig. 5. (a) Typical microchannel heat sink (MCHS) element and (b) finite volume mesh.

dition at the bottom is  $q'' = C$ ; adiabatic boundary conditions are applied at all other sides of the walls,  $\frac{\partial T}{\partial n} = 0$ ; and  $T = T_{in}$  in the fluid inlet region.

### 2.2.1. Pure water flow

For comparison purposes, a steady global energy balance based on the temperature difference between the channel inlet and outlet was performed, i.e.,

$$\rho_f u_m c_p A_c (T_{out} - T_{in}) = q'' \cdot A_{bottom} \quad (12)$$

Fig. 6 depicts the simulated water temperature rise between the channel inlet and outlet as a function of inlet Reynolds number and the theoretical values predicted by Eq. (12). In both of the two situations (scenarios, i.e., constant and temperature-dependent water properties), the simulation results match the energy balance predictions. The temperature difference between inlet and outlet are slightly higher when the water properties are dependent on the temperature especially for small Reynolds numbers. The reason is that water has a smaller thermal capacity at elevated fluid temperatures.

For noncircular conduits the channel aspect ratio or aspect factor (see Eq. (7a,b)) has a profound influence on the friction coefficient (or Poiseuille number):

$$C_f = f Re = \frac{\Delta p \cdot D_h^2}{2\mu_f u L} \quad (13)$$

which has a constant value of 16 for circular conduits. As shown in Fig. 7, the computed friction factor shows a good

agreement with the empirical correlation of Wu and Cheng (2003) for fully develop flow, i.e.,

$$f Re = 11.43 + 0.80 \exp(2.67 W_b / W_t) \quad (14)$$

For channel flow, the friction factor should be constant in the laminar flow regime. Indeed, the simulation result is almost parallel to the experimental empirical value within an error less than 3%. The simulation results for the temperature-dependent case are also within a 3% error margin. When the Reynolds number is larger than 1000, the theoretical friction factor hardly changes, i.e., the temperature differences in the fluid are small because of the relatively high mean velocities.

### 2.2.2. Nanofluid flow with heat transfer

In order to investigate the heat transfer characteristics of nanofluid flow in the trapezoidal microchannel, a CuO–water combination was used as the working fluid. Clearly, the thermal fluid properties of the nanofluid have to be updated. Thus, in the governing Eqs. (9) and (10), the following expressions were introduced, replacing the previous  $\mu_f$  and  $k_f$  parameters. Typically, for a very dilute suspension, the effective viscosity, density and specific heat capacity have the following forms (Xuan and Roetzel, 2000):

$$\mu_{eff} = \mu_f \frac{1}{(1 - \varphi)^{2.5}} \quad (15)$$

$$\rho_{eff} = \varphi \rho_p + (1 - \varphi) \rho_f \quad (16)$$

$$(\rho c_p)_{eff} = \varphi (\rho c_p)_p + (1 - \varphi) (\rho c_p)_f \quad (17)$$

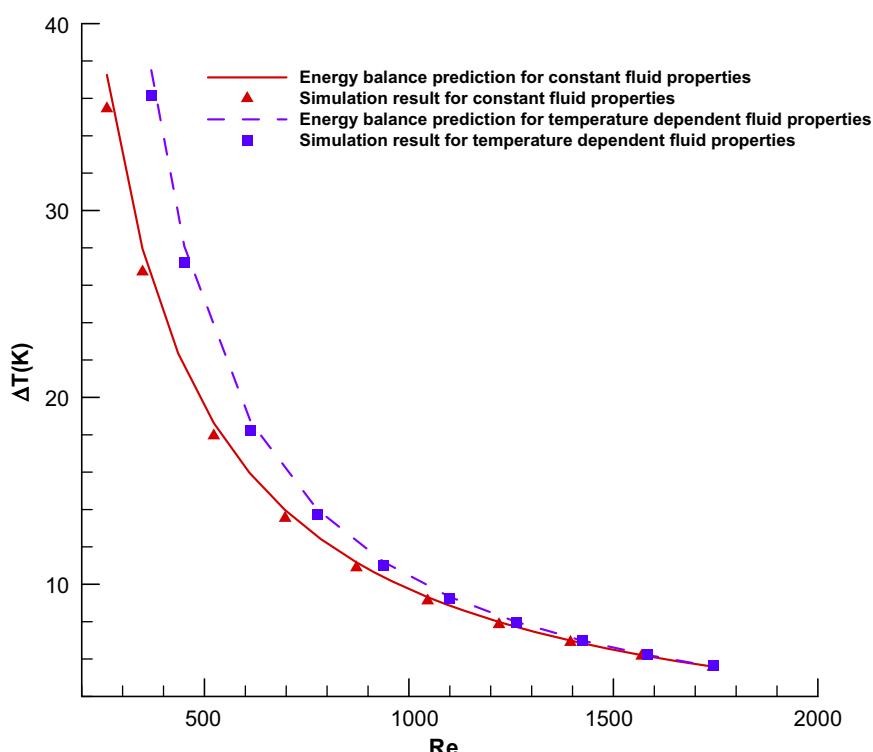


Fig. 6. Model validations: temperature rise from heat sink inlet to outlet.

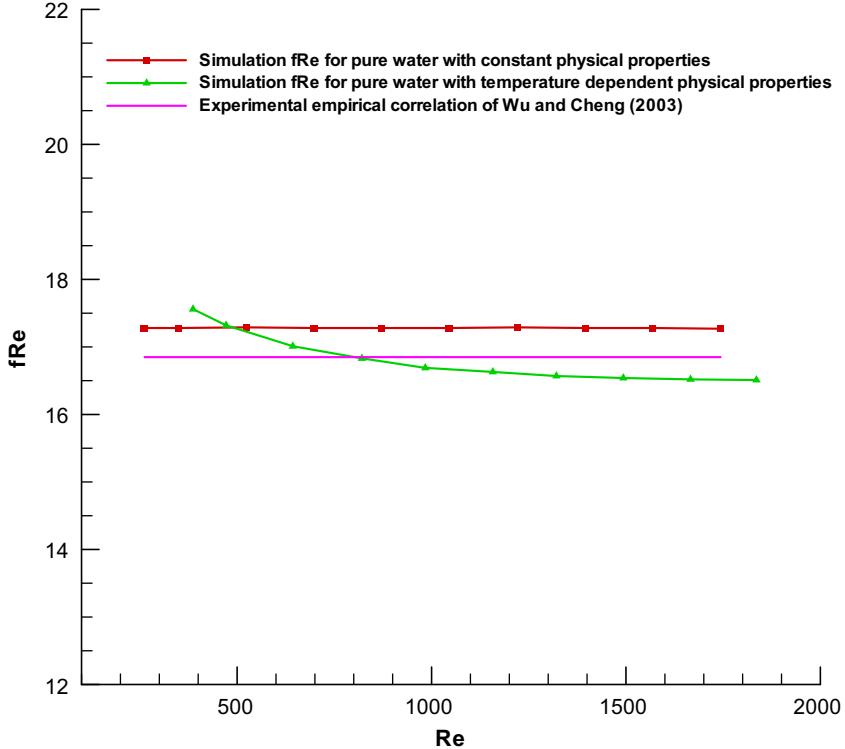


Fig. 7. Model validations: friction factors vs. Reynolds number.

here  $\rho_{\text{eff}}$  is the nanofluid density,  $\mu_{\text{eff}}$  is the nanofluid viscosity,  $(\rho c_p)_{\text{eff}}$  is the nanofluid specific heat capacity. For the effective thermal conductivity, the new KKL model (see Section 2.1) is employed.

The calculated fully-developed pressure gradient results for pure water and the CuO–water mixture at different volume fractions as a function of Reynolds number are given in Fig. 8. For the three cases, the Reynolds numbers differ even for same fluid entrance velocity, because of the different densities and dynamic viscosities. As expected, the trend shows a minor pressure gradient increase for the same Reynolds numbers when employing nanofluids. Specifically, the pressure gradient enhancement is less than 2% and 5% for CuO–water nanofluid with a volume fraction of 1% and 4%, respectively. Fig. 9 shows the pressure gradient increase at different mean velocities. The enhancement is less than 5% for CuO–water with a volume fraction of 1%, while for CuO–water with a volume fraction of 4% the increase is up to 15%.

Pumping power is needed to drive the working fluid in microchannels, which is defined as the product of the pressure drop across the channel ( $\Delta p$ ) and volumetric flow rate ( $Q$ ), i.e.,

$$P = \Delta p \cdot Q \quad (18)$$

As is shown in Fig. 10, there is not much difference in pressure drops when using pure water and nanofluids. There is an average 2% increase for CuO–water with a 1% volume fraction and an average 8% enhancement for the CuO–water with a 4% volume fraction. Thus, it does

not need require additional pumping power to drive nanofluid flow, especially for lower particle volume fractions. Chein and Chuang (2005) showed similar phenomena based on theoretical models and experimental correlations.

In order to investigate heat transfer enhancement for nanofluids, the average Nusselt numbers for different volume fractions were compared. The average Nusselt number is defined as

$$Nu_{\text{ave}} = \frac{q'' \cdot D_h}{(T_{w,\text{ave}} - T_{f,\text{ave}})k_f} \quad (19)$$

where  $T_{w,\text{ave}}$  is the surface-averaged temperature at the fluid–solid interface of area  $A_{\text{int}}$ ,  $T_{f,\text{ave}}$  is the volume-averaged temperature of the fluid field. Fig. 11 compares the average Nusselt number for pure water flow and CuO–water nanofluid flow with different volume fractions. It demonstrates that nanofluids can improve the thermal performance of microchannels: (i) the larger the volume fraction of nanoparticles, the higher is the thermal performance; (ii) a volume fraction of 1% CuO–water nanofluid shows an average 15% enhancement of thermal performance, when the volume fraction increases to 4%, there is a 20% increase over that of pure water.

In order to compare the thermal performance of nanofluids, the thermal resistance is employed:

$$\theta = \frac{T_{w,\text{ave}} - T_{\text{in}}}{q} \quad (20)$$

where  $T_{\text{in}}$  is the fluid inlet temperature and  $q$  is the heat added to the microchannel. As shown in Fig. 12, the thermal

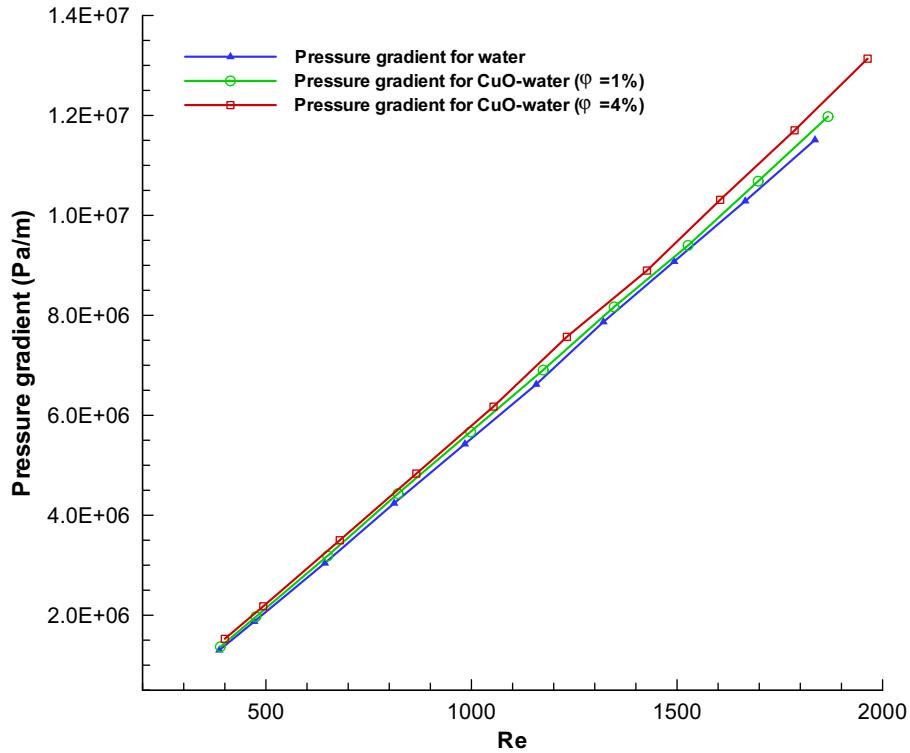


Fig. 8. Computational results: pressure gradient vs. Reynolds number.

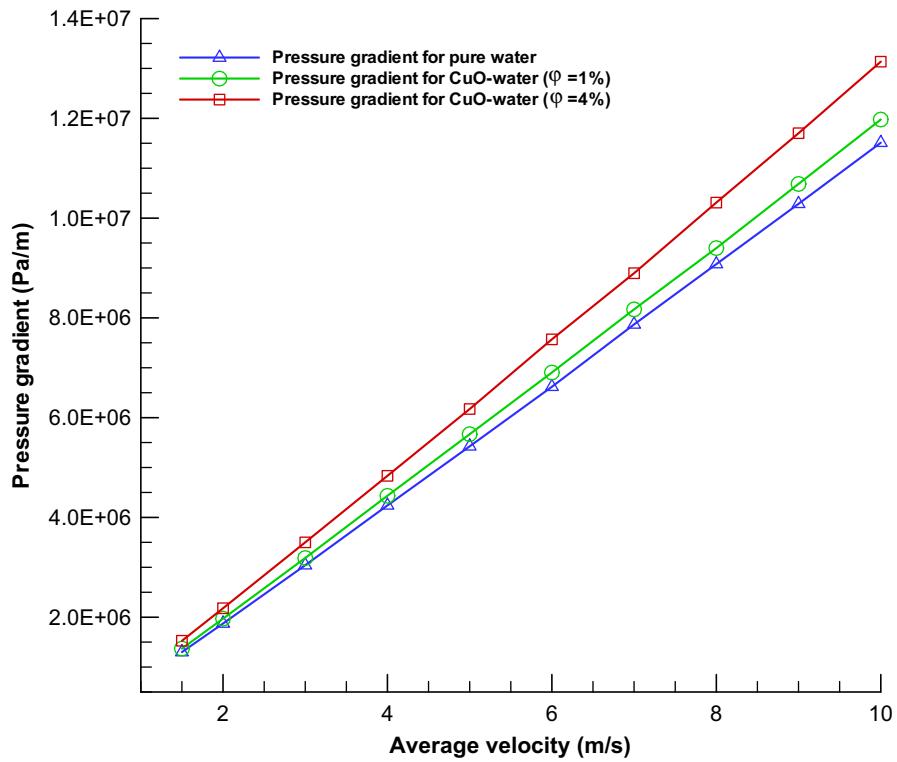


Fig. 9. Computational results: pressure gradient vs. mean velocity.

performance is enhanced when employing nanofluids. The average enhancement of thermal performance for CuO–

water with a volume fraction of 1% is about 11% and the increase is about 15% when the volume fraction is 4%.

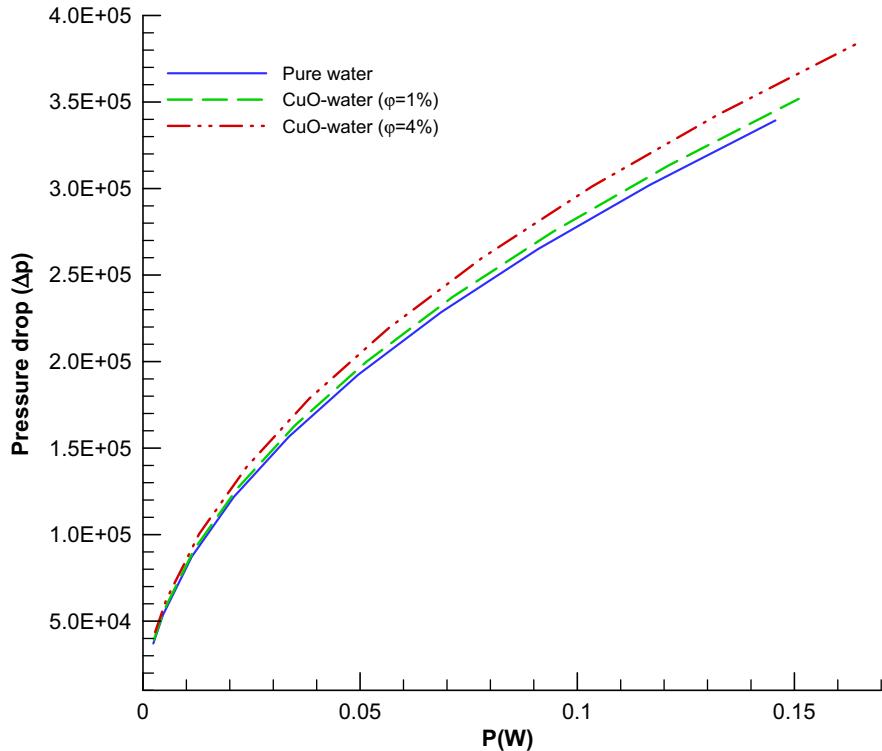


Fig. 10. Computational results: pressure drop vs. pumping power.

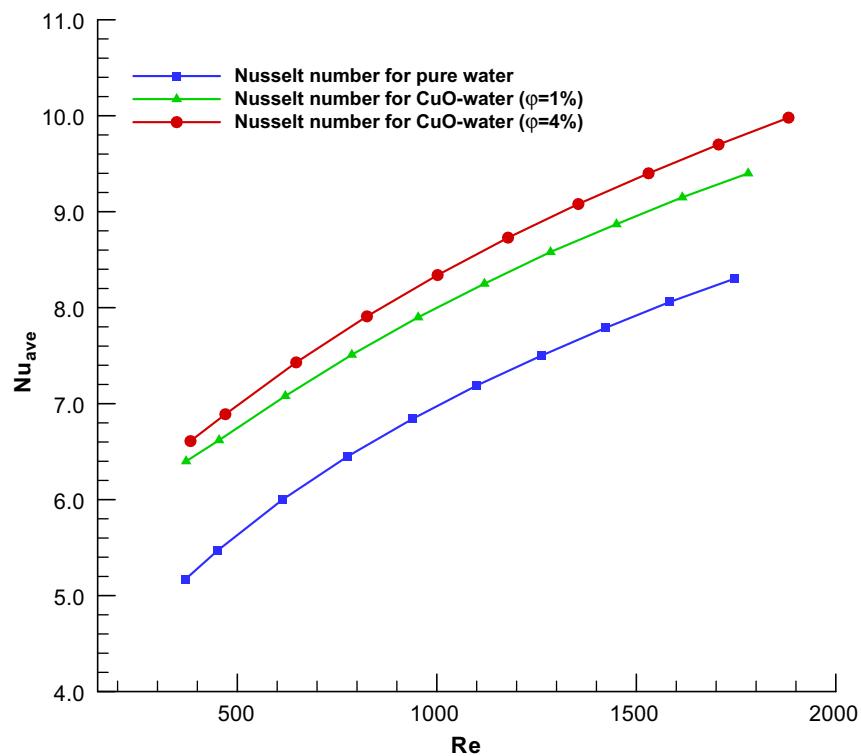


Fig. 11. Computational results: average Nusselt number vs. Reynolds number.

As depicted from Figs. 8, 10–12, nanofluids lower the thermal resistance and hence measurably enhance the per-

formance of microchannels with little pumping power added. In summary, microchannel heat sinks with nano-

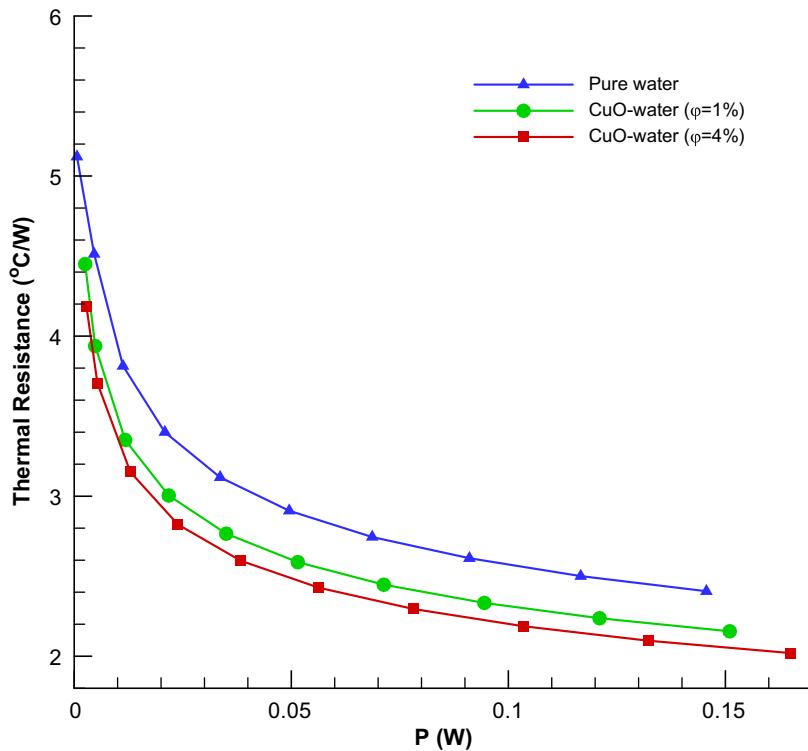


Fig. 12. Computational results: thermal resistance vs. pumping power.

fluids as coolant will be good candidates for the next generation of cooling devices.

### 3. Conclusions

Two effective thermal conductivity models for nanofluids were compared in detail, where the new KKL (Koo–Kleinstreuer–Li) model, based on Brownian-motion induced micro-mixing, achieved good agreements with the currently available experimental data sets. Employing the commercial Navier–Stokes solver CFX-10 (Ansys Inc., Canonsburg, PA) and user-supplied pre- and post-processing software, the thermal performance of nanofluid flow in a trapezoidal microchannel was analyzed using pure water and CuO–water with volume fractions of 1% and 4%. The results show that nanofluids do measurably enhance the thermal performance of microchannel mixture flow with a small increase in pumping power. The thermal performance increases with volume fraction; but, the extra pressure drop, or pumping power, will somewhat decrease the beneficial effects. Microchannel heat sinks with nanofluids are expected to be good candidates for the next generation of cooling devices.

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